

# Online Appendix to “Market Structure, Investment, and Technical Efficiencies in Mobile Telecommunications”

Jonathan T. Elliott,<sup>\*</sup> Georges V. Houngebbonon,<sup>†</sup>  
Marc Ivaldi,<sup>‡</sup> Paul T. Scott<sup>§</sup>

## B Demand Estimation Details

### B.1 Contraction Mapping

Here we consider an alternative version of the [Berry, Levinsohn and Pakes \(1995\)](#) (BLP) contraction mapping in which we observe market shares at the product-market level for Orange products but only aggregate firm-level market shares for the other products. We first show in section [B.1.2](#) that if we observe market shares at the firm-market level, the problem can be rewritten in such a way that the BLP contraction mapping proof holds. In section [B.1.3](#) we extend this result to the nested logit setting. Finally, in section [B.1.4](#) we show that if we observe some firm market shares only at the aggregate level (as is our case), the problem can still be rewritten to fit into the BLP contraction mapping proof setup.

#### B.1.1 Standard BLP Contraction Mapping Setup

We will start with the standard BLP setting in order to introduce notation. In this setting, there are products  $j \in \mathcal{J} = \{1, \dots, J\}$ , and we observe market shares  $\varsigma_{jm}$  for each product. We can express an individual’s utility for a product as  $u_{ijm} = \delta_{jm} + \mu_{ijm} + \varepsilon_{ijm}$ , which yields the type-specific market shares

$$s_{ijm} = \frac{\exp(\delta_{jm} + \mu_{ijm})}{\sum_{j'} \exp(\delta_{j'm} + \mu_{ij'm})}.$$

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<sup>\*</sup>Johns Hopkins University, [jonathan.elliott@jhu.edu](mailto:jonathan.elliott@jhu.edu)

<sup>†</sup>IFC-World Bank Group, [ghoungbonon@ifc.org](mailto:ghoungbonon@ifc.org), [gvivienh@gmail.com](mailto:gvivienh@gmail.com)

<sup>‡</sup>Toulouse School of Economics, [marc.ivaldi@tse-fr.eu](mailto:marc.ivaldi@tse-fr.eu)

<sup>§</sup>New York University, [ptscott@stern.nyu.edu](mailto:ptscott@stern.nyu.edu)

Aggregate market shares are given by

$$s_{jm}(\delta) = \int \frac{\exp(\delta_{jm} + \mu_{ijm})}{\sum_{j'} \exp(\delta_{j'm} + \mu_{ij'm})} dF(\mu_m).$$

The existence of the contraction mapping implies that there is a unique vector  $\delta$  such that  $s_m(\delta) = \varsigma_m$  for any observed vector of shares  $\varsigma_m$ .

### B.1.2 Grouped Products Extension

Our setting is one in which market shares are observed only for certain groupings of products. That is, let  $\mathcal{J}$  be partitioned into subsets  $\mathcal{J}_f$  with  $f \in \mathcal{F} = \{1, 2, \dots, F\}$ . For each  $f$ , we observe only the market share  $\varsigma_{ft}$  for all the products within  $\mathcal{J}_f$ . The subsets  $\mathcal{J}_f$  may include individual products (i.e., in our application each Orange product would have its own  $\mathcal{J}_f$  set) or several products (i.e., each non-Orange firm has one  $\mathcal{J}_f$  group that includes all that firm's products).

Providing a parametric form, let  $\delta_{jm} = \theta_1 x_{jm} + \xi_{jm}$ , where  $\theta_1$  would capture what is often referred to as "linear parameters," i.e., parameters that can typically be estimated outside of the contraction mapping because they only shift the mean utility component  $\delta_{jm}$  that the contraction mapping aims to recover. In this extension, the  $\theta_1$  parameters must be included in the contraction mapping.

We cannot recover  $\delta_{jm}$  (or  $\xi_{jm}$ ) separately for different  $j \in \mathcal{J}_f$ . We assume  $\xi_{jm} = \xi_{fm}$  for all  $j \in \mathcal{J}_f$  for each  $f$ .

Let  $\bar{x}_{fm}$  be the mean value of  $x_{fm}$  for those products within  $\mathcal{J}_f$ . Then, we have  $\delta_{jm} = \theta_1 \bar{x}_{fm} + \theta_1 x_{jm}^d + \xi_{fm}$ , where  $x_{jm}^d := x_{jm} - \bar{x}_{fm}$ . We define  $\tilde{\delta}_{fm} = \theta_1 \bar{x}_{fm} + \xi_{fm}$ , and  $\tilde{\mu}_{ijm} = \theta_1 x_{jm}^d + \mu_{ijm}$ . This very nearly allows us to re-define the model in terms where we could apply the original BLP proof strategy to establish the contraction mapping. The only problem is that  $\tilde{\mu}_{ijm}$  is defined over  $j$ , where we would need it to be defined over  $f$  in order to apply the same proof strategy. Let's consider the aggregation over  $j$  to  $f$ :

$$s_{ifm}(\tilde{\delta}) = \sum_{j \in \mathcal{J}_f} \frac{\exp(\tilde{\delta}_{fm} + \tilde{\mu}_{ijm})}{\sum_{j' \in \mathcal{J}} \exp(\tilde{\delta}_{f(j')m} + \tilde{\mu}_{ij'm})},$$

where  $f(j')$  refers to the  $f$  associated with product  $j'$ .

Defining  $\tilde{\mu}_{ifm} = \log \left( \sum_{j \in \mathcal{J}_f} \exp(\tilde{\mu}_{ijm}) \right)$ , it follows that

$$\sum_{j \in \mathcal{J}_f} \exp(\tilde{\delta}_{fm} + \tilde{\mu}_{ijm}) = \exp(\tilde{\delta}_{fm} + \tilde{\mu}_{ifm}),$$

and therefore

$$s_{ifm}(\tilde{\delta}) = \sum_{j \in \mathcal{J}_f} \frac{\exp(\tilde{\delta}_{fm} + \tilde{\mu}_{ijm})}{\sum_{f'} \exp(\tilde{\delta}_{f'm} + \tilde{\mu}_{if'm})}.$$

We can then aggregate up to market-level shares  $s_{fm}$  by integrating over the  $\tilde{\mu}_{ifm}$ , and we have rewritten our extended setting in a way that allows us to apply the BLP proof strategy.

### B.1.3 Grouped Products Extension with Nested Logit

In the more general random coefficients nested logit (RCNL) model introduced by [Grigolon and Verboven \(2014\)](#) (henceforth, GV), we can construct analogous formulas that will allow us to recover group-specific mean demands  $\tilde{\delta}$ .

In the RCNL model, type-specific market shares are as follows:

$$s_{ijm} = \frac{\exp\left(\frac{\delta_{jm} + \mu_{ijm}}{1-\sigma}\right) \exp(I_{ig(j)})}{\exp\left(\frac{I_{ig(j)}}{1-\sigma}\right) \exp(I_i)},$$

where  $\sigma \in [0, 1)$  is the nesting parameter,  $g(j)$  return the nest to which  $j$  belongs,<sup>1</sup> and

$$\begin{aligned} I_{ig} &= (1 - \sigma) \log \left( \sum_{j \in \mathcal{J}_g} \exp\left(\frac{\delta_{jm} + \mu_{ijm}}{1-\sigma}\right) \right), \\ I_i &= \log \left( 1 + \sum_{g \in \mathcal{G}} \exp(I_{ig}) \right). \end{aligned}$$

In this extension, we redefine  $\tilde{\delta}_{fm}$  and  $\tilde{\mu}_{ifm}$  to incorporate  $\sigma$ . Let  $\tilde{\delta}_{fm} = \frac{\theta_1 \bar{x}_{fm} + \xi_{fm}}{1-\sigma}$ ,  $\tilde{\mu}_{ijm} = \frac{\theta_1 x_{jm}^d + \mu_{ijm}}{1-\sigma}$ , and  $\tilde{\mu}_{ifm} = \log \left( \sum_{j \in \mathcal{J}_f} \exp(\tilde{\mu}_{ijm}) \right)$ . Then

$$s_{ifm} = \frac{\exp(\tilde{\delta}_{fm} + \tilde{\mu}_{ifm}) \exp(I_{ig(f)})}{\exp\left(\frac{I_{ig(f)}}{1-\sigma}\right) \exp(I_i)}$$

where  $I_{ig} = (1 - \sigma) \log \left( \sum_{f \in \mathcal{F}_g} \exp(\tilde{\delta}_{fm} + \tilde{\mu}_{ifm}) \right)$  and  $\mathcal{F}_g = \{f \in \mathcal{F} : g(f) = g\}$ .

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<sup>1</sup>We will assume that products produced by the same firm belong to the same group. Formally, for each  $f$ ,  $g(j) = g_f$  for all  $j \in \mathcal{J}_f$ .

GV note that, substituting in our notation,

$$f(\tilde{\delta}) = \tilde{\delta} + \log(\varsigma) - \log(s(\tilde{\delta}))$$

is a contraction mapping if

$$1 - \frac{1}{s_f} \frac{\partial s_f}{\partial \tilde{\delta}_f} \geq 0.$$

Unlike in GV, this holds in our case. Explicitly,

$$\frac{\partial s_f}{\partial \tilde{\delta}_f} = \left(1 - \frac{\sigma}{1 - \sigma} s_{f|g} - s_f\right) s_f,$$

and so

$$1 - \frac{1}{s_f} \frac{\partial s_f}{\partial \tilde{\delta}_f} = \frac{\sigma}{1 - \sigma} s_{f|g} + s_f \geq 0 \quad \Leftrightarrow \quad \sigma s_{f|g} + (1 - \sigma) s_f \geq 0.$$

This condition holds for all  $\sigma \in [0, 1)$ .

#### B.1.4 Market Aggregation Extension

In our setting we observe market shares only at the aggregate level for some firms. We assume in this extension  $\xi_{jm} = \xi_{f(j)}$  for all  $j, m$  and recover  $\xi_f$  for each  $f$ . We will proceed in this section using the non-nested setting introduced in section B.1.2, but the results hold using the analogues to the RCNL expressions introduced in section B.1.3.

Analogous to the previous setup, let  $\bar{x}_f$  be the mean value of  $x_{jm}$  across products  $j \in \mathcal{J}_f$  and markets  $m$ ,  $\bar{x}_f = \frac{1}{MJ_f} \sum_m \sum_{j \in \mathcal{J}_f} x_{jm}$ . Then,  $\delta_{jm} = \theta_1 \bar{x}_{f(j)} + \theta_1 x_{jm}^d + \xi_{f(j)}$ , where we now define  $x_{jm}^d := x_{jm} - \bar{x}_{f(j)}$ . Analogously defining  $\tilde{\delta}_f = \theta_1 \bar{x}_f + \xi_f$ ,  $\tilde{\mu}_{ijm} = \theta_1 x_{jm}^d + \mu_{ijm}$ , and  $\tilde{\mu}_{ifm} := \log\left(\sum_{j \in \mathcal{J}_f} \exp(\tilde{\mu}_{ijm})\right)$ , then

$$\bar{s}_{if}(\tilde{\delta}) = \sum_m w(m) \frac{\exp(\tilde{\delta}_f + \tilde{\mu}_{ifm})}{\sum_{f'} \exp(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm})}.$$

We can aggregate up to aggregate firm shares  $\bar{s}_f$  by integrating over  $\tilde{\mu}_{ifm}$ :

$$\bar{s}_f = \int \sum_m w(m) \frac{\exp(\tilde{\delta}_f + \tilde{\mu}_{ifm})}{\sum_{f'} \exp(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm})} dF(\tilde{\mu}_{ifm}) = \int \frac{\exp(\tilde{\delta}_f + \tilde{\mu}_{ifm})}{\sum_{f'} \exp(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm})} dG(\tilde{\mu}_{ifm}).$$

The final expression makes clear that the BLP contraction mapping proof strategy still holds in this aggregate setting.

Since we observe product-level market shares for every market for Orange products, we allow  $\xi_{jm}$  to differ by product and market for all  $j \in \mathcal{J}_{ORG}$ .

When coding the contraction mapping, we follow [Conlon and Gortmaker \(2020\)](#) in implementing the SQUAREM algorithm ([Varadhan and Roland, 2008](#)).

## B.2 Calibration

In this section, we explain how we derived the values used from calibration.

First, we have the own price elasticity for Orange, defined in equation 10, which is an elasticity with respect to a proportional change in the prices of all of Orange's products at once. An equivalent way of writing this elasticity is

$$\begin{aligned}
 \frac{d \ln s_{ORG}}{d \ln P_{ORG}} &= \sum_{j \in \mathcal{J}_{ORG}} \frac{\partial s_{ORG}}{\partial P_j} \frac{P_j}{s_{ORG}} \\
 &= \sum_{j \in \mathcal{J}_{ORG}} \frac{P_j}{s_{ORG}} \sum_{j' \in \mathcal{J}_{ORG}} \frac{\partial s_{j'}}{\partial P_j} \\
 &= \sum_{j \in \mathcal{J}_{ORG}} \frac{\partial s_j}{\partial P_j} \frac{P_j}{s_j} \frac{s_j}{s_{ORG}} \sum_{j' \in \mathcal{J}_{ORG}} \frac{\partial s_{j'}}{\partial P_j} / \frac{\partial s_j}{\partial P_j} \\
 &= \sum_{j \in \mathcal{J}_{ORG}} e_j \frac{s_j}{s_{ORG}} \sum_{j' \in \mathcal{J}_{ORG}} -DIV^{j,j'},
 \end{aligned} \tag{1}$$

where  $d \ln P_{ORG}$  represents a proportional change in the prices of Orange's plans,  $e_j$  denotes product  $j$ 's own price elasticity and  $DIV^{j,j'}$  is the diversion ratio from product  $j$  to  $j'$ ,

$$DIV^{j,j'} = -\frac{\frac{\partial s_{j'}}{\partial P_j}}{\frac{\partial s_j}{\partial P_j}}.$$

The third line follows by dividing and multiplying by  $\frac{\partial s_j}{\partial P_j} / s_j$ .

In [Bourreau, Sun and Verboven \(2021\)](#), we take elasticities ( $e_j$ ) from Table A.4, diversion ratios ( $DIV^{j,j'}$ ) from Table A.3, and quantities (shares) from Table 3. Plugging in all these numbers, we find an elasticity of  $-2.36$ .

Next, we have Orange's diversion ratio to the outside option, defined as

$$DIV^{ORG,0} = -\frac{\frac{ds_0}{d \ln P_{ORG}}}{\frac{ds_{ORG}}{d \ln P_{ORG}}}, \tag{2}$$

where  $d \ln P_{ORG}$  represents a proportional change in all of Orange's prices. By the chain rule,

$$\frac{ds_0}{d \ln P_{ORG}} = \sum_{j \in \mathcal{J}_{ORG}} \frac{\partial s_0}{\partial P_j} P_j.$$

We then have

$$\frac{ds_0}{d \ln P_{ORG}} = \sum_{j \in \mathcal{J}_{ORG}} DIV^{j,0} \frac{\partial s_j}{\partial P_j} P_j = \sum_{j \in \mathcal{J}_{ORG}} DIV^{j,0} e_j s_j, \quad (3)$$

where  $DIV^{j,0}$  is the diversion ratio from  $j$  to the outside option.

Turning to the denominator of equation 2, we can write

$$\frac{ds_{ORG}}{d \ln P_{ORG}} = e_{ORG} s_{ORG}. \quad (4)$$

Finally, we substitute equations 3 and 4 into equation 2:

$$DIV^{ORG,0} = \frac{\sum_{j \in \mathcal{J}_{ORG}} DIV^{j,0} e_j s_j}{e_{ORG} s_{ORG}}. \quad (5)$$

Taking elasticities, diversion ratios, and shares from the same sources in BSV as above, this yields a diversion ratio of 0.036.

Being precise, the moment we use for estimation averages over market-level diversion ratios, while the diversion ratio calculated here is based on objects that are already sample averages. That is, there is a difference in whether we average before or after taking the ratio expressed in equation 5. However, this difference appears to have a trivial impact on the value of the diversion ratio. In our sample at our parameter estimates, the difference in the diversion ratio if averaging is done before taking the ratio versus if done after is 0.00087.

## C Data Appendix

This appendix provides additional description of our main datasets and variables. Section C.1 presents the characteristics of mobile tariffs and the tariff dataset. Section C.2 describes the measurement of the quality of mobile data. Section C.3 discusses network sharing in France in 2015.

### C.1 Product Data

#### C.1.1 Product Characteristics

We collect data on mobile phone plans released between November 2013 and October 2015, along with their characteristics, from operators' quarterly catalogs. It includes postpaid plans from the four MNOs and the largest MVNO (EI Telecom) as well as their prepaid

plans.<sup>2</sup> Promotional plans, typically released during summer and around Christmas, are not included in the dataset.

Plan characteristics include tariff, voice and data limits, handset subsidy, length of commitment, and whether or not plans were bundled with fixed services. As described in section 2.2, we choose representative mobile-only plans for each firm and adjust monthly prices based on contract duration and handset subsidies.

We take over 100 contracts from catalogs, and from them we construct 21 representative products in our model's choice set. We define categories of plans according to their level of data limits: less than 500 MB, 500–3 000 MB, 3 000–7 000 MB and more than 7 000 MB. These thresholds are chosen following discussions with industry experts and the statistical distribution of chosen plans. The second data limit category—that is, contracts with 500–3 000 MB—we have further split according to their voice allowances: unlimited or not, making a total of five categories of phone plans. Low data limit plans typically do not have unlimited voice, and high data limit contracts typically come with unlimited voice allowance, so we do not split these categories by the voice limit. We exclude plans bundled with fixed broadband or television.

We choose the least expensive plan in each category as the category's representative plan. Some customers keep old plans that are no longer available, so we fill these missing data by using the most similar representative plan. While some plans with handset subsidies have corresponding standalone versions, some do not. We adjust the prices of these latter plans using data on the price of handsets and the upfront payment required by Orange. We collect these data for both iPhone and Samsung, the two most popular handsets. We then distribute the handset cost over 24 months and update the monthly plan price by subtracting off the monthly cost of the handset. In addition, we assume that Orange's handset subsidies apply to other operators' subsidized contracts because we do not observed their upfront costs.

### **C.1.2 Soft Data Limits**

For plans with data limits, the download speed is reduced for usage above allowance if no add-on is purchased. The maximal download speed under throttling is typically 128 Kbps. With this download speed, it would take over half-an-hour to download a 30 MB file, compared to 2 minutes under a theoretical non-throttled speed of 2 Mbps in a 3G network, and 24 seconds given a moderate 4G download speed of 10 Mbps. Basically, only emails and light

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<sup>2</sup>ORG's contracts include not only those that are sold through its main brand, but also others sold under alternative brands such as SOSH, BNP Paribas Mobile, FNAC Mobile, Click Mobile, Carrefour Mobile, etc.

web pages can be opened under throttling. As presented in Table 1 below, this download speed is not always specified by operators in their contracts. When it is, it may depend on the location of the usage (local or abroad). The actual download speed experienced by customers is a function of the number of simultaneous users, its location and handset. In our demand model, however, we assume that any data consumption over the data limit yields a speed of exactly 128 Kbps.

Table 1: Maximal Download Speed under Throttling (Kbps)

Operator	National	Roaming
ORG	128*	ns
SFR	ns	ns
BYG	128	32
FREE	ns	ns

\*:except video streaming.  
ns  $\equiv$  not specified.  
Source: operators' contracts

## C.2 Quality Data

Quality measures are constructed using download speed test results provided by Ookla. Test results come from users who use Ookla's free Internet speed test, called "Speedtest," using a web browser or within an app. Using speed tests in France in the second quarter of 2016 yields 1 056 285 individual speed tests. Each speed test records the download speed, mobile network operator, and the user's location. We aggregate speed tests by averaging measured download speeds over tests for a given operator and geographic market, yielding an operator-market quality measure. An operator-market quality measure is, on average, an average of 284 test results. Note that our estimates rely on an instrument for these quality measures (see section 3.2.2), alleviating concerns about attenuation bias.

## C.3 Network Sharing

Network sharing occurs when a network operator shares a part or the whole of its network resources with a retail competitor. These resources can be passive network elements, such as antenna supports, masts, or active network elements, such as frequency bandwidths. Passive network sharing affects coverage differentiation but not necessarily quality differentiation. It



typically consists of operators sharing the same tower and potentially the cost of electricity. In general, it is any agreement between MNOs that do not involve the sharing of available frequency bandwidths.

In contrast, under active network sharing (Radio Access Network-Sharing), operators cannot differentiate in terms of quality, defined as the frequency bandwidth available per customer. Typically, it consists of the sharing of frequency bands and the network elements involved in data transmission. Roaming agreements, whereby an operator's customers rely on the network of a host operator to communicate, is the highest level of active network sharing. It does not offer any possibility for quality or coverage differentiation.

Table 2 below presents the network sharing agreements reached between 2012 and 2015. These agreements apply to two types of areas according to their population density. "White Areas" or "Zones Blanches" correspond to areas where population density is so low that network deployment by several operators is not profitable. These areas, which are typically rural, are designated by the regulator and represent roughly 1% of the population and 10% of the national surface. Only ORG, SFR and BYG have invested in these areas.

The most widespread network technologies in the White Areas are 2G, EDGE and GPRS.<sup>3</sup> However, 3G technology has been recently deployed. As of the end of December 2015, half of ORG and BYG's networks in these areas were covered by 3G, compared to 35% for SFR. In general, only one operator invests in a given White Area, and 64% of antennas in these areas are involved in a roaming agreement. Rival operators roam over the network of the only operator that invests in the area. As a result, there is no quality differentiation. For the remaining 36% of antennas, operators share passive network elements.

At the national level, FREE's customers can roam over ORG's 2G and 3G networks as long as there is no FREE antenna nearby. As a result, FREE cannot differentiate from ORG on 2G and 3G technologies, except when a FREE antenna is nearby its customer. In addition, FREE does not have access to networks in Zones Blanches where BYG or SFR is the leader. MVNOs have roaming agreements with their hosts and therefore cannot differentiate in terms of quality or coverage.

Our model focuses on high-density areas to avoid the need to explicitly model network sharing. During our period of study, the only active network sharing in such areas would have involved FREE's customers receiving data from 2G and 3G infrastructure owned and operated by ORG. Meanwhile, ORG and FREE each owned and operated their own distinct

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<sup>3</sup>EDGE and GPRS are suitable for low-speed mobile data services.

4G network infrastructure, At the margin, 4G investments were how firms were differentiating and competing in download speeds in 2015.

Table 2: Network Sharing Agreements 2012-2015

		FREE	ORG	SFR	BYG
Zone Blanche	Roaming: 64% of 2G & 3G antenna			↔	
	Passive sharing: 36% of antenna			↔	
Low Density	2G and 3G RAN-Sharing	✗	✗		↔
	4G Roaming	✗	✗		→
High Density		✗	✗	✗	✗
National	Passive sharing			↔	
	2G and 3G Roaming		→	✗	✗

Source: Summary from discussions with ORG's experts.

Note: ↔: two-way (reciprocal) sharing,  $A \rightarrow B$  one-way sharing hosted by operator B.

## D Supplementary Results

### D.1 Alternative Cost Specification

In this section we consider the robustness of our counterfactual results to the specification of infrastructure costs. In the main text, we use a specification in which base station costs are proportional to bandwidth. In this section, we consider the impact on our results of an alternative specification in which we assume that all of the infrastructure costs are fixed per base station. That is, we replace the infrastructure cost function specification (equation 22) with

$$C_{fm}(R_{fm}, B_{fm}) = c_{fm}^s \frac{A_m}{A(R_{fm})}.$$

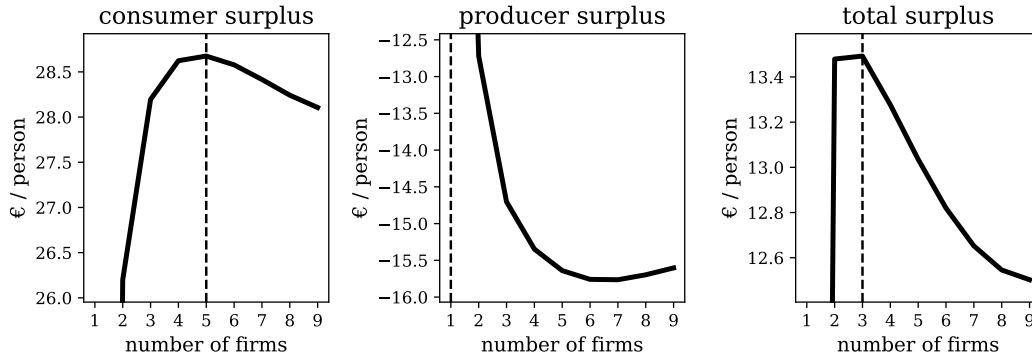
We use this specification of infrastructure costs to recover  $\hat{c}_{fm}^s$  for each firm  $f$  and market  $m$ , and these parameters now have the interpretation of costs per base station (rather than costs per base station per unit of bandwidth).<sup>4</sup>

At four firms, base station costs are the same for both cost specifications – this is just the average cost per base station recovered from the data. At fewer firms, base station costs are cheaper in this specification where costs do not scale with bandwidth; at more firms, base stations are more expensive in this specification.

<sup>4</sup>Each  $\hat{c}_{fm}^s$  is simply  $B_{fm}\tilde{c}_{fm}^s$ , where  $\tilde{c}_{fm}^s$  is the infrastructure cost parameter recovered using the specification used in the main text.

Figure 1 plots different measures of welfare as we change the number of symmetric firms, analogous to figure 9 in section 6.1.

Figure 1: Counterfactual Welfare



*Note:* Welfare is measured in euros per capita relative to monopoly.

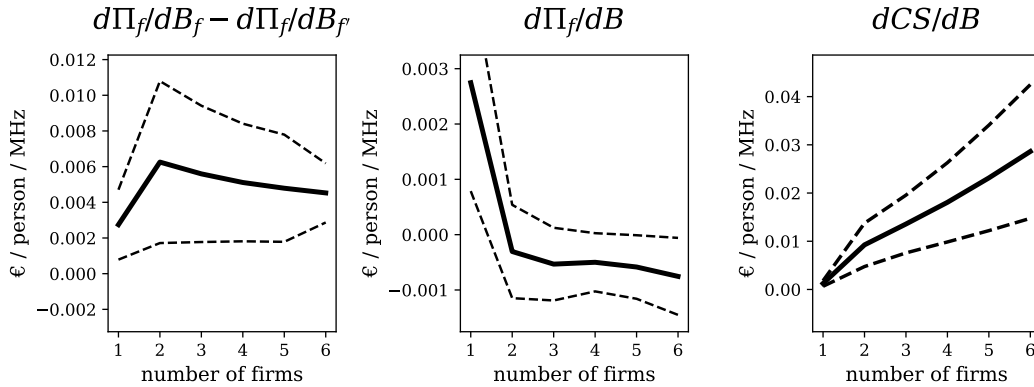
Relative to the case in which costs are proportional to bandwidth, this cost specification implies fewer firms maximize both consumer and total surplus. However, this specification probably overstates the extent of scale efficiencies.

With this cost specification, there is the introduction of another source of economies of scale from the duplication of fixed costs. Thus, in this case, if we hold the number of base stations per firm fixed, more firms means more base stations, which means higher costs. In contrast, when base station costs are proportional to bandwidth, if we hold the number of base stations per firm fixed, then total base station costs do not change as we change the number of firms, given that the total bandwidth in the industry is fixed. While base stations certainly involve some fixed costs, firms can and do avoid duplicative fixed costs by engaging in passive network sharing, in which firms share some base station infrastructure (such as the land or the tower). Thus, we view as unrealistic these counterfactuals in which base stations involve fixed costs that are unavoidably replicated as we increase the number of firms. In contrast, our main cost specification is consistent with an equilibrium in which firms co-locate their base stations and share fixed costs.

The results for this alternative specification point to the importance of scale efficiencies that can be attained without integration. One might wonder in particular whether the gains from economies of pooling, which play an important role in our main results, can be attained without consolidation. We note that such gains would require firms from sharing their active network infrastructure (also known as Radio Access Network (RAN) sharing). Such network sharing is rare, while the sharing of passive infrastructure is common. This may be because firms do not find it profitable to share their active network infrastructure as in [Fund et al.](#)

(2017). Active network sharing undermines firm's incentives to differentiate on quality. An open question is whether there is a possible regulatory framework that would allow firms to attain efficiencies from pooling network infrastructure without undermining incentives to invest.

Figure 2: Bandwidth Derivatives



*Note:* Derivatives are evaluated at the symmetric equilibrium values. The derivative of own profits with respect to another firm's bandwidth ( $d\Pi_f/dB_{f'}$ ) is undefined in the monopoly case. In the first subplot, therefore, what is reported in the case of only one firm is simply the derivative of own profits with respect to own bandwidth ( $d\Pi_f/dB_f$ ). Dashed lines represent 95% confidence intervals.

We also assessed the marginal value of spectrum with this alternative cost specification. Results were similar: marginal surplus exceed firms' willingness to pay by a factor of four instead of the factor of five we saw with the main specification in section 6.2.

## D.2 Equilibrium without Path Loss

Here we show that in symmetric equilibria the optimal number of base stations per consumer is constant with respect to population density when there is no path loss or interference.

Let  $N_{fm}$  represent the number of base stations operated by operator  $f$  in municipality  $m$ . The number of consumers within each cell is given by  $\frac{D_m A_m}{N_{mf}}$ , where  $D_m$  is the population density and  $A_m$  is the municipality's area. We now rewrite equation 19 as

$$Q_{fm} = \bar{Q}_{fm} - \frac{D_m A_m}{N_{mf}} q^D (\mathbf{P}_{fm}, \mathbf{Q}_{fm}, \mathbf{P}_{-fm}, \mathbf{Q}_{-fm}), \quad (6)$$

where  $q^D (\mathbf{P}_{fm}, \mathbf{Q}_{fm}, \mathbf{P}_{-fm}, \mathbf{Q}_{-fm})$  represents equilibrium data consumption per capita. Note that channel capacity per base station  $\bar{Q}_{fm}$  is exogenous without path loss and interference. Bandwidth is endowed, so there are no choice variables to influence channel capacity. The firm's only infrastructure choice here is effectively how many consumers they want to

serve with each base station.

Consider firm  $f$ 's variable profit function, equation 21, now written in per-consumer terms and as a function of quality:

$$\Pi_{fm}^V(\mathbf{P}_f, \mathbf{Q}_{fm}) \equiv (\mathbf{P}_f - \mathbf{c}_f^u) \cdot \mathbf{s}_f(\mathbf{P}_{fm}, \mathbf{Q}_{fm}, \mathbf{P}_{-fm}, \mathbf{Q}_{-fm}).$$

Let  $\lambda_{fm} = \frac{D_m}{N_{fm}}$ , and note that  $\lambda_{fm}$  can represent the firm's infrastructure choice variable. Rewrite variable profits as

$$\Pi_{fm}^V(\mathbf{P}_f, \lambda_{fm}) \equiv (\mathbf{P}_f - \mathbf{c}_f^u) \cdot \mathbf{s}_f(\mathbf{P}_{fm}, \lambda_{fm}, \mathbf{P}_{-fm}, \boldsymbol{\lambda}_{-fm}),$$

noting that the share function can be expressed as a function of  $\lambda_{fm}$  since delivered download speeds are determined by the congestion equation 6, and here  $\lambda_{fm} = \frac{D_m}{N_{fm}}$  defines the congestion equation above.

Given the cost function expressed in equation 22, infrastructure costs are  $c_{fm}^s B_{fm} N_{fm}$ , and costs per capita can be expressed as

$$c_{fm}^s B_{fm} \frac{N_{fm}}{D_m A_m} = c_{fm}^s B_{fm} \lambda_{fm}^{-1} A_m^{-1}.$$

Both variable profits and infrastructure costs depend on population density  $D_m$  and the number of base stations  $N_{fm}$  only through their ratio  $\lambda_{fm} = \frac{D_m}{N_{fm}}$ . Therefore, the firm's optimum and the equilibrium level of investment entail a value for  $\lambda$ , or a number of base stations per consumer. Therefore, when we do comparative statics with respect to population density, the equilibrium number of base stations will be proportional to population density.

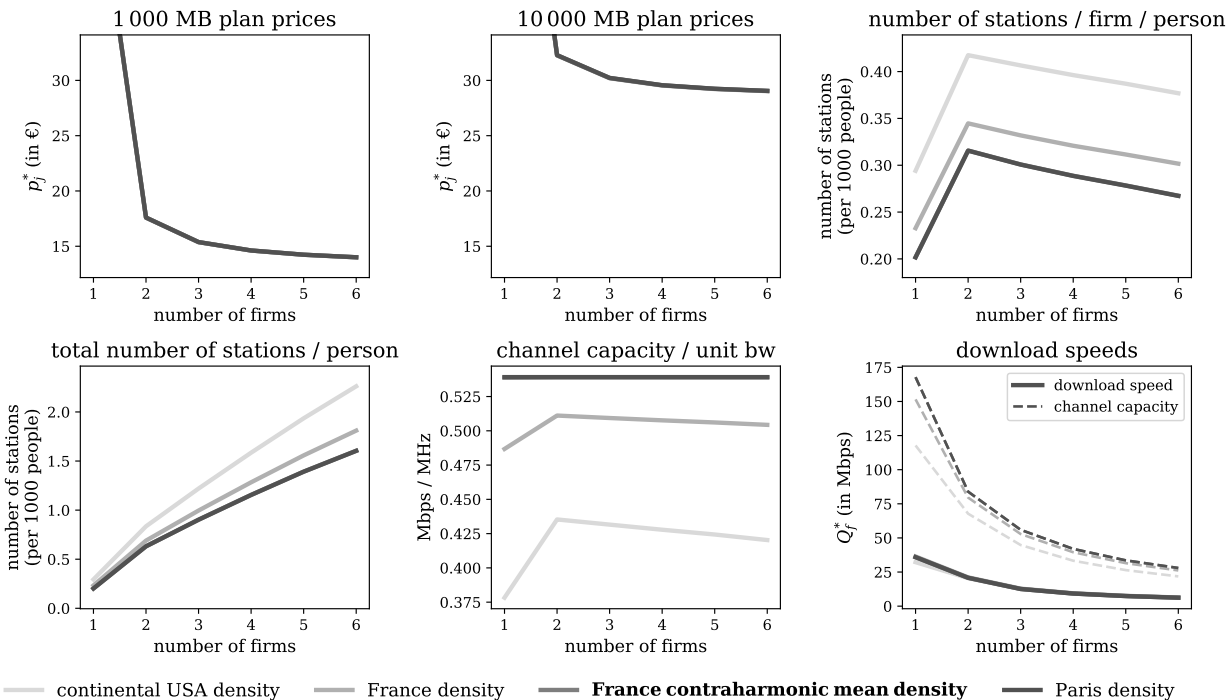
### D.3 Impact of Population Density

Our main counterfactual simulations consider a market with moderate population density. This density of 2 792 persons / km<sup>2</sup> roughly corresponds to a high-density suburb. A natural question is whether the population density affects the trade-off between market power and scale efficiencies, perhaps changing the optimal number of firms. We first note that, without path loss, the equilibrium comparative statics with respect to population density would be very straightforward.

As shown above, without path loss, channel capacity is fixed by the bandwidth owned and operated by the firm. The cell radius will not affect channel capacity. The decision of cell

radius amounts to a decision of how many customers to serve with each base station, with the firm effectively choosing the optimal level of congestion. The population density will not affect this choice when we think about it in terms of the optimal number of consumers per base station (or the optimal level of congestion). As population density increases, the optimal number of consumers per station remains constant, implying base station area will be inversely proportional to population density. Equilibrium outcomes like prices and delivered download speeds remain the same. See section D.2 for a more formal account.

Figure 3: Counterfactual Prices and Qualities by Density



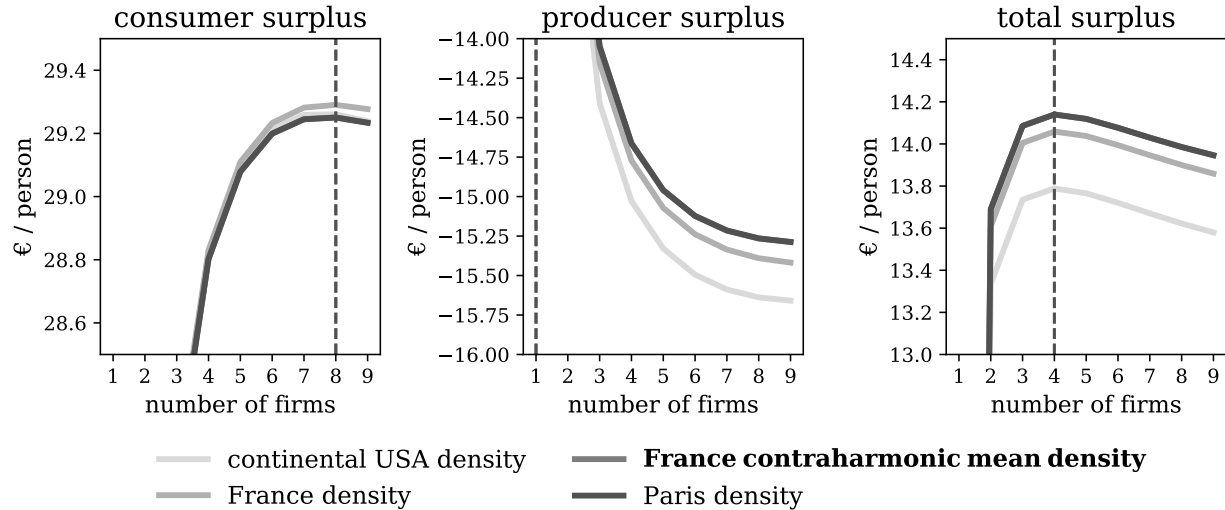
*Note:* Each line in a subplot corresponds to a different population density, with the darker the line, the higher the density. Channel capacity is per base station. Download speeds are the average speed of transmission received by a user, including wait times.

In addition to France's population-weighted mean population density (2 792 people/km<sup>2</sup>), we consider three alternative population densities: the raw population densities of the continental USA (43.1) and France (123.9)—note that these are both quite low densities as both countries involve large unpopulated areas—and the population density of Paris (20 588).

Figure 3 illustrates how equilibrium outcomes for these different population densities. Certain outcomes are indeed affected by population density. Naturally, path loss is more severe when serving a less dense market, demonstrated by lower channel capacities per unit of bandwidth in Figure 3 (despite higher levels of investment per person).<sup>5</sup>

<sup>5</sup>For each of these densities, we use the Hata model of path loss presented in Appendix A.1.1. This Hata

Figure 4: Counterfactual Welfare by Density



*Note:* Depicted are measures of welfare as a function of number of firms. Each line in a subplot corresponds to a different population density, with the darker the line, the higher the density. Welfare is measured in euros per capita relative to monopoly, so for each plot the value at 1 firm is 0. Dashed vertical lines denote the number of firms that maximizes that measure of welfare.

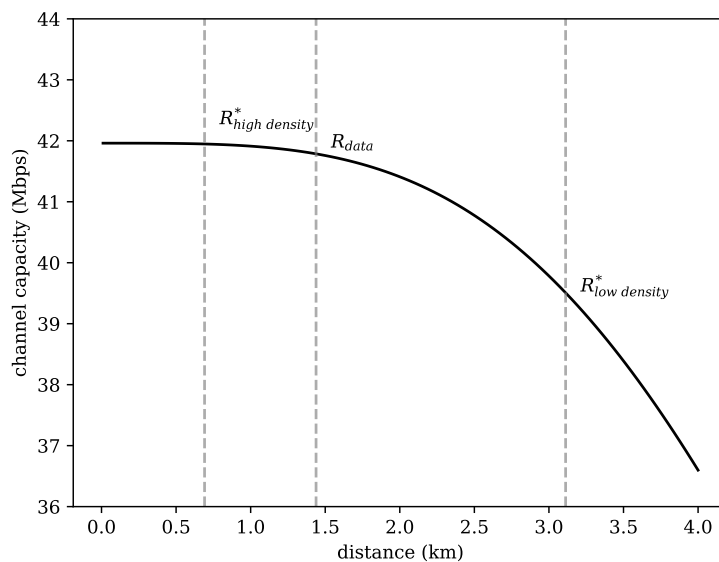
Otherwise, the comparative statics with respect to population density are very similar to what we would expect without path loss. In other words, we do not see substantial economies of density. Figure 5 depicts channel capacity as a function of the cell's radius. For radii in the range of the equilibrium radii in our counterfactuals, this function is quite flat, which is consistent with economies of density not being substantial at these population densities, although they may be at extremely low densities.

The optimal number of firms (for consumer or total surplus), depicted in Figure 4, is quite robust to the population density. Equilibrium outcomes like prices and delivered download speeds are extremely similar for different population densities. A takeaway is that, given the equilibrium cell sizes we observe, economies of density only appear to be a significant concern in very sparsely populated areas.

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model is for small cities. We have also simulated these counterfactual densities with rural and suburban Hata models of path loss for the associated densities, which exhibit less path loss as a function of distance. Results look similar but correspond more closely to the case of no path loss (in which the density does not matter).

Figure 5: Channel Capacity as Function of Radius



Note:  $R_{data}$  corresponds to the average radius of a cell in our data.  $R^*_{low density}$  and  $R^*_{high density}$  correspond to the equilibrium radius chosen in the four-firm equilibrium when the market has, respectively, a density of France and a density equal to the population-weighted mean population density of France. Bandwidth is set equal to the same total bandwidth as in the rest of our counterfactuals divided by four (for four firms), and spectral efficiency is also set to the same value as in the rest of our counterfactuals.



Table 3: Notation

Symbol	Description
$f$	indexes firms
$i$	indexes consumers
$j$	indexes mobile phone plans
$\mathcal{J}$	set of mobile phone plans
$\ell$	indexes a location
$\mathcal{L}(R)$	set of locations within hexagon of radius $R$
$m$	indexes markets (municipalities)
$\gamma_m$	data transmission efficiency in market $m$
$\varepsilon_{ij}$	idiosyncratic, consumer-plan-level demand shock
$\theta$	demand parameters
$\theta_{pi}$	price coefficient
$\theta_{p0}$	parameter controlling the mean of the price coefficient
$\theta_{pz}$	parameter controlling the heterogeneity in the price coefficient
$\theta_v$	coefficient on dummy for unlimited voice
$\theta_O$	average Orange demand shock
$\theta_c$	opportunity cost of time spent downloading data coefficient
$\theta_{di}$	parameter of exponential distribution that defines distribution from which a consumer's utility of data consumption is drawn
$\theta_{d0}$	parameter controlling the mean of $\theta_{di}$
$\theta_{dz}$	parameter controlling the heterogeneity in $\theta_{di}$
$\vartheta_i$	random shock to consumer's utility of data consumption, distributed exponentially with parameter $\theta_{di}$
$\boldsymbol{\theta}_i$	vector containing $\theta_{pi}$ and $\theta_{di}$
$\xi_{jm}$	market-level demand shock
$\sigma$	nesting parameter
$B_{fm}$	bandwidth (in Megahertz)
$c_j^u$	cost per user
$c_{fm}^s$	cost per base station and unit of bandwidth
$\bar{d}_j$	data consumption limit of phone plan $j$
$D_m$	population density
$F$	used for CDFs
$H$	number of hours in a month
$I_\ell(R_{fm})$	interference power at location $\ell$ when cell radius is $R_{fm}$
$N_{fm}$	number of base stations for firm $f$ in market $m$
$q_{m\ell}$	data transmission speed at location $\ell$ in municipality $m$ (in Mbits/second)
$\bar{Q}_{fm}$	channel capacity (in Mbits/second)
$Q_{fm}$	download speed (in Mbits/second) of firm $f$ in market $m$
$Q^L$	throttled download speed (in Mbits/second)
$Q_{fm}^D$	demand requests (in Mbits/second)
$P_j$	price of phone plan $j$
$R_{fm}$	radius of area served by one base station (in km)
$S_\ell$	signal power at location $\ell$
$s_{jm}$	market share
$\mathbf{s}$	vector of market shares
$u$	utility of a phone plan
$v_j$	dummy variable for whether plan $j$ has an unlimited voice allowance
$w$	utility from data consumption over course of month
$x$	monthly data consumption
$z_i$	consumer $i$ 's income

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